

MATH 3060 Tutorial 5

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October 12, 2022

1. Let $C^\infty[0, 1]$ be the set of smooth functions on $[0, 1]$.

(a) Is the function

$$f \mapsto \left(g : [0, 1] \ni x \mapsto \int_0^x f(t) dt \right)$$

continuous with respect to d_∞ ?

(b) Is the function

$$f \mapsto f'$$

continuous with respect to d_∞ ?

2. (a) Let X be a nonempty set, denote $B(X)$ the set of bounded real-valued functions on X . Show that d_∞ defines a complete metric on $B(X)$.
- (b) Let d be a metric on X , show that the subset $C_0(X) \subset B(X)$ consisting of bounded continuous functions is closed.

3. Show that

$$\|fg\|_r \leq \|f\|_p \|g\|_q$$

when

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{r}.$$

4. Let $f : X \rightarrow Y$ be a map of metric spaces. Show that the followings are equivalent:
- (a) f is continuous.
- (b) For any $B \subset Y$, $f^{-1}(B^\circ) \subset f^{-1}(B)^\circ$.
- (c) For any $A \subset X$, $f(\overline{A}) \subset \overline{f(A)}$.